Problem 7

Find all values of a such that f is continuous on $\mathbb{R}$:

$$f(x) = \begin{cases} x + 1 & \text{if } x \le a \\ x^2 & \text{if } x > a \end{cases}$$

Solution

$x + 1$ and x^2 are polynomial functions, which are continuous everywhere. Consequently, any points of discontinuity can only occur at the endpoint of the intervals on which these functions are defined. Require the function to be continuous at this endpoint.

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$

$$\lim_{x \to a}(x + 1) = \lim_{x \to a} x^2$$

$$a + 1 = a^2$$

Solve for a.

$$a^2 - a - 1 = 0$$

$$a = \frac{1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)}$$

$$a = \frac{1 \pm \sqrt{5}}{2}$$

$$a = \left\{ \frac{1 - \sqrt{5}}{2}, \frac{1 + \sqrt{5}}{2} \right\}$$